Ch.3 Image Transforms

- **Introduction.**
- A classification of image transforms.
  - Point transforms.
  - Local transforms.
  - Global transforms.
- Processing in frequency domain.
- Geometrical transformations.
Introduction

- Image processing can be defined to include all operations that manipulate image data.
  - Manipulate pixel values.
  - Transform an image from one representation into an alternative.

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Classification of Image Transforms

- Point transforms
  - modify individual pixels
  - modify pixels’ locations

- Local transforms
  - Output derived from neighbourhood
Classification of Image Transforms

- Global transforms
  - Whole image contributes to each output value

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Classification of Image Transforms

- **Point transforms**
  - modify individual pixels
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### Grey Scale Manipulation
- Brightness adjustment
- Contrast adjustment

### Thresholding

### Histogram manipulation
- equalisation
Point Transforms

› Grey Scale Manipulation
  ▪ Brightness adjustment (Add a constant to all values)
    \[ g' = g + k \]
    \[(k = 50)\]

Point Transforms

› Grey Scale Manipulation
  ▪ Contrast modifications (Scale all values by a constant)
    \[ g' = g \times k \]
    \[(k = 1.5)\]
Point Transforms

- Thresholding
  - A simple segmentation technique that is very useful for scenes with solid objects resting on a contrasting background.
  - All pixels above a determined (threshold) grey level are assumed to belong to the object, and all pixels below that level are assumed to be outside the object.

Thresholding

- E.g) Transform grey/colour image to binary
  \[ \text{if } f(x, y) > T \text{ output } = 1 \]
  \[ \text{else } 0 \]

- How to find $T$?
  - In our course:
  - Manual User defines a threshold
Point Transforms

› Histogram

• Histogram: is a graph showing the number of pixels in an image at each different intensity value found in that image.

For an 8-bit gray scale image there are 256 different possible intensities, and so the histogram will graphically display 256 numbers showing the distribution of pixels amongst those gray scale values.
Point Transforms

- Histogram Equalization (Manipulation)
  - Modify distribution of grey values to achieve some effect

Equalisation/Adaptive Equalisation

- Specifically to make histogram uniform
**Equalisation Transform**

- Equalised image has \( n \times m/l \) pixels per grey level
- Cumulative to level \( j \)
  - \( jm/l \) pixels
- Equate to a value in input cumulative histogram \( C[i] \)
  - \( C[i] = jm/l \)
  - \( j = C[i]l/nm \)

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Local Transforms

- Convolution
- Applications
  - smoothing
  - sharpening
  - matching

Convolution Definition

\[ g'(r,c) = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} g(r-x,c-y) \cdot t(x,y) \]

Place template on image
Multiply overlapping values in image and template
Sum products and normalise
(Templates usually small)
### Example

<table>
<thead>
<tr>
<th>Image</th>
<th>Template</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>...  3 5 7 4 4 ...</td>
<td>...  1 1 1 ...</td>
<td>...  6 6 6 ...</td>
</tr>
<tr>
<td>...  4 5 8 5 4 ...</td>
<td>...  1 2 1 ...</td>
<td>...  6 7 6 ...</td>
</tr>
<tr>
<td>...  4 6 9 6 4 ...</td>
<td>...  1 1 1 ...</td>
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*Divide by template sum*

### Separable Templates

- Convolve with \( n \times n \) template
  - \( n^2 \) multiplications and additions
- Convolve with two \( n \times 1 \) templates
  - \( 2n \) multiplications and additions
Example

- Laplacian template
  
  \[
  \begin{matrix}
  0 & -1 & 0 \\
  -1 & 4 & -1 \\
  0 & -1 & 0 \\
  \end{matrix}
  \]

- Separated kernels
  
  \[
  \begin{matrix}
  -1 \\
  -1 & 2 & -1 & 2 \\
  -1 \\
  \end{matrix}
  \]
Composite Filters

- Convolution is distributive
  \[ A \otimes (B \otimes C) = (A \otimes B) \otimes C \]
- Can create a composite filter and do a single convolution

- The result obtained by convolving the image \( A \), with \( B \) \( n \) multiplications and additions and convolving the mediate result with \( C \) (a further \( n \) multiplication and divisions) is identical to what is obtained by the original method.
Applications

- Usefulness of convolution is the effects generated by changing templates
  - Smoothing
    - Noise reduction
  - Sharpening
    - Edge enhancement
  - Template matching
    - Image Registration

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  - **Smoothing**
    - Noise reduction
  - **Sharpening**
    - Edge enhancement
  - **Template matching**
    - A later lecture

Smoothing Spatial Filters

- One of the simplest convolution operations we can perform is a smoothing operation
  - Simply average all of the pixels in a neighbourhood around a central value
  - Especially useful in removing noise from images
  - Also useful for highlighting gross detail

<table>
<thead>
<tr>
<th>1/9</th>
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Simple averaging filter
The image at the top left is an original image of size 500*500 pixels. The subsequent images show the image after filtering with an averaging filter of increasing sizes: 3, 5, 9, 15 and 35. Notice how detail begins to disappear.

Image Smoothing Example

Image Smoothing Example

Image Smoothing Example

Weighted Smoothing Filters

More effective smoothing filters can be generated by allowing different pixels in the neighbourhood different weights in the averaging function:

- Pixels closer to the central pixel are more important.
- Often referred to as a weighted averaging.

<table>
<thead>
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<th>1/16</th>
<th>2/16</th>
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<td>1/16</td>
<td>2/16</td>
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By smoothing the original image we get rid of lots of the finer detail which leaves only the gross features for thresholding.


Another Smoothing Example

- By smoothing the original image we get rid of lots of the finer detail which leaves only the gross features for thresholding.

Original Image | Smoothed Image | Thresholded Image
Filtering is often used to remove noise from images.
Sometimes a median filter works better than an averaging filter.
Averaging Filter Vs. Median Filter Example

Strange Things Happen At The Edges!

At the edges of an image we are missing pixels to form a neighbourhood.
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Sharpening

- What is it?
  - Enhancing discontinuities
  - Edge detection
- Why do it?
  - Perceptually important
  - Computationally important

Edge Definition

An edge is a significant local change in image intensity.
Edge Types

- Step edge

- Line edge
Edge Types

- Step edge

- Line edge

- Roof edge

First Derivative, Gradient Edge Detection

- If an edge is a discontinuity
- Can detect it by differencing
Roberts Cross Edge Detector

-1 0
0 1
0 -1
1 0

- Simplest edge detector
- Inaccurate localisation

Prewitt/Sobel Edge Detector

-1 -1 -1
0 0 0
1 1 1

-1 0 1
-1 0 1
-1 0 1
-1 0 1

Image Processing and Computer Vision
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  - **Global transforms.**
    - Processing in frequency domain.

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Global Transforms

- Computing a new value for a pixel using the whole image as input
- Cosine and Sine transforms
- Fourier transform
  - Frequency domain processing
Cosine/Sine

- A halfway solution to the Fourier Transform
- Used in image coding

\[
DCT(i, j) = \sqrt{2N} C(i) C(j) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} pixel(x, y) \cos \left( \frac{(2x+1)j\pi}{2N} \right) \cos \left( \frac{(2y+1)j\pi}{2N} \right)
\]

\[
C(x) = \begin{cases} 
\frac{1}{\sqrt{2}} & \text{if } x = 0 \\
1 & \text{otherwise}
\end{cases}
\]

Jean Baptiste Joseph Fourier

Fourier was born in Auxerre, France in 1768

- Most famous for his work “La Théorie Analitique de la Chaleur” published in 1822

Nobody paid much attention when the work was first published.
One of the most important mathematical theories in modern engineering.
Fourier

- All periodic signals can be represented by a sum of appropriately weighted sine/cosine waves

\[
F_n = \begin{bmatrix}
W_{0,0} & \cdots & W_{0,N-1} \\
\vdots & \ddots & \vdots \\
W_{N-1,0} & \cdots & W_{N-1,N-1}
\end{bmatrix} \begin{bmatrix}
f_0 \\
\vdots \\
f_{N-1}
\end{bmatrix}
\]

and

\[
W_{n,j} = \frac{1}{\sqrt{N}} e^{-j2\pi \frac{n}{N}}
\]

The Discrete Fourier Transform (DFT)

The Discrete Fourier Transform of \( f(x, y) \), for \( x = 0, 1, 2…M-1 \) and \( y = 0, 1, 2…N-1 \), denoted by \( F(u, v) \), is given by the equation:

\[
F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi \frac{ux}{M} + \frac{vy}{N}}
\]

for \( u = 0, 1, 2…M-1 \) and \( v = 0, 1, 2…N-1 \).
DFT & Images

Spatial Domain
Frequency Domain

The Inverse DFT

It is really important to note that the Fourier transform is completely reversible. The inverse DFT is given by:

\[
f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(ux/M + vy/N)}
\]

for \(x = 0, 1, 2\ldots M-1\) and \(y = 0, 1, 2\ldots N-1\)
**Frequency Domain Filtering**

› Convolution Theorem:

> Convolution in spatial domain is equivalent to Multiplication in frequency domain

---

**The DFT and Image Processing**

To filter an image in the frequency domain:

1. Compute \( F(u,v) \) the DFT of the image
2. Multiply \( F(u,v) \) by a filter function \( H(u,v) \)
3. Compute the inverse DFT of the result
Smoothing is achieved in the frequency domain by dropping out the high frequency components. The basic model for filtering is:

\[ G(u,v) = H(u,v)F(u,v) \]

where \( F(u,v) \) is the Fourier transform of the image being filtered and \( H(u,v) \) is the filter transform function. Low pass filters – only pass the low frequencies, drop the high ones.
The transfer function for the ideal low pass filter can be given as:

\[ H(u, v) = \begin{cases} 
1 & \text{if } D(u, v) \leq D_0 \\
0 & \text{if } D(u, v) > D_0 
\end{cases} \]

where \( D(u, v) \) is given as:

\[ D(u, v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2} \]